

Scotogenic A_4 Neutrino Model for Nonzero θ_{13} and Large δ_{CP}

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Abstract

Assuming that neutrinos acquire radiative seesaw Majorana masses through their interactions with dark matter, i.e. scotogenic from the Greek 'scotos' meaning darkness, and using the non-Abelian discrete symmetry A_4 , we propose a model of neutrino masses and mixing with nonzero θ_{13} and necessarily large leptonic CP violation, allowing both the normal and inverted hierarchies of neutrino masses, as well as quasi-degenerate solutions.

In 2006, a one-loop mechanism was introduced [1] linking neutrino mass with dark matter. The idea is very simple. The standard model of particle interactions is extended to include a second scalar doublet (η^+, η^-) which is odd under an exactly conserved Z_2 symmetry [2], as well as three neutral fermion singlets N_i which are also odd under Z_2 . This requirement immediately allows the possibility of having the real (or imaginary) part of η^0 as a dark-matter candidate, which was first pointed out also in Ref. [1]. As shown in Fig. 1, this results in the radiative generation of seesaw Majorana neutrino masses from dark matter, i.e. scotogenic from the Greek 'scotos' meaning darkness.

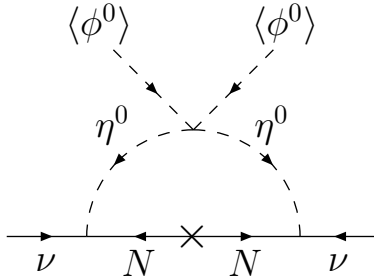


Figure 1: One-loop generation of scotogenic Majorana neutrino mass.

The non-Abelian discrete symmetry A_4 was introduced [3, 4, 5] to achieve the seemingly impossible, i.e. the existence of a lepton family symmetry consistent with the three very different charged-lepton masses m_e , m_μ , m_τ . It was subsequently shown [6] to be a natural theoretical framework for neutrino tribimaximal mixing, i.e. $\sin^2 \theta_{23} = 1$, $\tan^2 \theta_{12} = 0.5$, and $\theta_{13} = 0$. This pattern was consistent with experimental data until recently, when the Daya Bay Collaboration reported [7] the first precise measurement of θ_{13} , i.e.

$$\sin^2 2\theta_{13} = 0.092 \pm 0.016(\text{stat}) \pm 0.005(\text{syst}), \quad (1)$$

followed shortly [8] by the RENO Collaboration, i.e.

$$\sin^2 2\theta_{13} = 0.113 \pm 0.013(\text{stat}) \pm 0.019(\text{syst}). \quad (2)$$

This means that tribimaximal mixing is not a good description, and more importantly, leptonic CP violation is now possible because $\theta_{13} \neq 0$, just as hadronic CP violation in the quark sector is possible because $V_{ub} \neq 0$.

Recently, it was shown [9] that A_4 is still a good symmetry for understanding this pattern, using a new simple variation of the original idea [6]. In that proposal, neutrinos acquire Majorana masses through their direct interactions with Higgs triplets. We study here instead the corresponding scenario with the radiative mechanism of Fig. 1.

The symmetry A_4 is that of the even permutation of four objects. It has twelve elements and is the smallest group which admits an irreducible three-dimensional representation. Its character table is given below. The basic multiplication rule of A_4 is

$$\underline{3} \times \underline{3} = \underline{1} + \underline{1}' + \underline{1}'' + \underline{3} + \underline{3}. \quad (3)$$

As first shown in Ref. [3], for $(\nu_i, l_i) \sim \underline{3}$, $l_i^c \sim \underline{1}, \underline{1}', \underline{1}''$, and $\Phi_i = (\phi_i^0, \phi_i^-) \sim \underline{3}$, the charged-

class	n	h	χ_1	$\chi_{1'}$	$\chi_{1''}$	χ_3
C_1	1	1	1	1	1	3
C_2	4	3	1	ω	ω^2	0
C_3	4	3	1	ω^2	ω	0
C_4	3	2	1	0	0	-1

Table 1: Character table of A_4 .

lepton mass matrix is given by

$$\mathcal{M}_l = \begin{pmatrix} v_1 & 0 & 0 \\ 0 & v_2 & 0 \\ 0 & 0 & v_3 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix} \begin{pmatrix} f_1 & 0 & 0 \\ 0 & f_2 & 0 \\ 0 & 0 & f_3 \end{pmatrix}, \quad (4)$$

where $v_i = \langle \phi_i^0 \rangle$ and $\omega = e^{2\pi i/3} = -1/2 + i\sqrt{3}/2$. For $v_1 = v_2 = v_3 = v/\sqrt{3}$, we then obtain

$$\mathcal{M}_l = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix} \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}, \quad (5)$$

where $m_e = f_1 v$, $m_\mu = f_2 v$, $m_\tau = f_3 v$. The original A_4 symmetry is now broken to the residual symmetry Z_3 , i.e. lepton flavor triality [10], with $e \sim 1$, $\mu \sim \omega^2$, $\tau \sim \omega$. This is a good symmetry of the Lagrangian as long as neutrino masses are zero. Exotic scalar decays are predicted and may be observable at the Large Hadron Collider (LHC) in some regions of parameter space [11, 12].

To obtain nonzero neutrino masses, we assign $\eta \sim \underline{1}$ and $N_i \sim \underline{3}$ under A_4 . We also add the scalar singlets $\sigma_i \sim \underline{3}$ with nonzero $\langle \sigma_i \rangle$. The resulting 3×3 Majorana mass matrix for N_i is then

$$\mathcal{M}_N = \begin{pmatrix} A & F & E \\ F & A & D \\ E & D & A \end{pmatrix}, \quad (6)$$

which is the analog of

$$\mathcal{M}_\nu = \begin{pmatrix} a & f & e \\ f & a & d \\ e & d & a \end{pmatrix}, \quad (7)$$

considered in Ref. [9]. (A better way to enforce Eq. (6) is to postulate gauged $B - L$ and assume complex neutral scalars which transform as $\underline{1}$, $\underline{3}$ under A_4 , in complete analogy to the scalar triplets of Ref. [9].) Instead of enforcing $E = F = 0$ which is required for tribimaximal mixing, we assume here that $F = -E$ which may be maintained by an interchange symmetry [6, 13].

Consider now the tribimaximal basis, i.e.

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}. \quad (8)$$

Since $\nu_{1,2,3}$ are connected to $N_{1,2,3}$ through the identity matrix, we find

$$\mathcal{M}_N^{(1,2,3)} = \begin{pmatrix} A + D & 0 & 0 \\ 0 & A & C \\ 0 & C & A - D \end{pmatrix}, \quad (9)$$

where $C = (E - F)/\sqrt{2} = \sqrt{2}E$.

The diagram of Fig. 1 is exactly calculable from the exchange of $\text{Re}(\eta^0)$ and $\text{Im}(\eta^0)$ and is given by [1]

$$(\mathcal{M}_\nu)_{ij} = \sum_k \frac{h_{ik}h_{jk}M_k}{16\pi^2} \left[\frac{m_R^2}{m_R^2 - M_k^2} \ln \frac{m_R^2}{M_k^2} - \frac{m_I^2}{m_I^2 - M_k^2} \ln \frac{m_I^2}{M_k^2} \right], \quad (10)$$

where $\sum_k h_{ik}(h_{jk})^* = |h|^2 \delta_{ij}$, and $m_{R,I}$ are the masses of $\sqrt{2}\text{Re}(\eta^0)$ and $\sqrt{2}\text{Im}(\eta^0)$, respectively. In the limit $m_R^2 - m_I^2 = 2\lambda_5 v^2$ is small compared to $m_0^2 = (m_R^2 + m_I^2)/2$, and $m_0^2 \ll M_k^2$, Eq. (10) reduces to

$$(\mathcal{M}_\nu)_{ij} = \frac{\lambda_5 v^2}{8\pi^2} \sum_k \frac{h_{ik}h_{jk}}{M_k} \left[\ln \frac{M_k^2}{m_0^2} - 1 \right]. \quad (11)$$

In the tribimaximal basis of Eq. (9), we then have

$$h_{ik} = h \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta e^{i\phi} \\ 0 & \sin \theta e^{-i\phi} & \cos \theta \end{pmatrix} \begin{pmatrix} e^{i\alpha'_1/2} & 0 & 0 \\ 0 & e^{i\alpha'_2/2} & 0 \\ 0 & 0 & e^{i\alpha'_3/2} \end{pmatrix}, \quad (12)$$

with

$$\begin{pmatrix} \cos \theta & \sin \theta e^{i\phi} \\ -\sin \theta e^{-i\phi} & \cos \theta \end{pmatrix} \begin{pmatrix} A & C \\ C & A - D \end{pmatrix} \begin{pmatrix} \cos \theta & -\sin \theta e^{-i\phi} \\ \sin \theta e^{i\phi} & \cos \theta \end{pmatrix} = \begin{pmatrix} e^{i\alpha'_2} M_2 & 0 \\ 0 & e^{i\alpha'_3} M_3 \end{pmatrix}. \quad (13)$$

The neutrino mixing matrix U has 4 parameters: s_{12}, s_{23}, s_{13} and δ_{CP} [14]. We choose the convention $U_{\tau 1}, U_{\tau 2}, U_{e3}, U_{\mu 3} \rightarrow -U_{\tau 1}, -U_{\tau 2}, -U_{e3}, -U_{\mu 3}$ to conform with that of the tribimaximal mixing matrix of Eq. (8), then

$$\mathcal{M}_\nu^{(1,2,3)} = U_{TB}^T U \begin{pmatrix} e^{i\alpha_1} m'_1 & 0 & 0 \\ 0 & e^{i\alpha_2} m'_2 & 0 \\ 0 & 0 & m'_3 \end{pmatrix} U^T U_{TB}, \quad (14)$$

where $m'_{1,2,3}$ are the physical neutrino masses, with

$$m'_2 = \sqrt{m_1'^2 + \Delta m_{21}^2}, \quad (15)$$

$$m'_3 = \sqrt{m_1'^2 + \Delta m_{21}^2/2 + \Delta m_{32}^2} \quad (\text{normal hierarchy}), \quad (16)$$

$$m'_3 = \sqrt{m_1'^2 + \Delta m_{21}^2/2 - \Delta m_{32}^2} \quad (\text{inverted hierarchy}). \quad (17)$$

We now diagonalize $\mathcal{M}_\nu^{(1,2,3)}$ using

$$U_\epsilon \mathcal{M}_\nu^{(1,2,3)} U_\epsilon^T = \begin{pmatrix} e^{i\alpha'_1 m'_1} & 0 & 0 \\ 0 & e^{i\alpha'_2 m'_2} & 0 \\ 0 & 0 & e^{i\alpha'_3 m'_3} \end{pmatrix}, \quad (18)$$

from which we obtain $U' = U_{TB} U_\epsilon^T$. To obtain U with the usual convention, we rotate the phases of the μ and τ rows so that $U'_{\mu 3} e^{-i\alpha'_3/2}$ is real and negative, and $U'_{\tau 3} e^{-i\alpha'_3/2}$ is real and positive. These phases are absorbed by the μ and τ leptons and are unobservable. We then rotate the $\nu_{1,2}$ columns so that $U'_{e1} e^{-i\alpha'_3/2} = U_{e1} e^{i\alpha''_1/2}$ and $U'_{e2} e^{-i\alpha'_3/2} = U_{e2} e^{i\alpha''_2/2}$, where U_{e1} and U_{e2} are real and positive. The physical relative Majorana phases of $\nu_{1,2}$ are then $\alpha_{1,2} = \alpha'_{1,2} + \alpha''_{1,2}$. The three angles and the Dirac phase are extracted according to

$$\tan^2 \theta_{12} = |U'_{e2}/U'_{e1}|^2, \quad \tan^2 \theta_{23} = |U'_{\mu 3}/U'_{\tau 3}|^2, \quad \sin \theta_{13} e^{-i\delta_{CP}} = U'_{e3} e^{-i\alpha'_3/2}. \quad (19)$$

The effective Majorana neutrino mass in neutrinoless double beta decay is then given by

$$m_{ee} = |U_{e1}^2 e^{i\alpha_1} m'_1 + U_{e2}^2 e^{i\alpha_2} m'_2 + U_{e3}^2 m'_3|. \quad (20)$$

In Eq. (9), let A be real and positive by convention, then both C and D may be complex, i.e. $C = C_R + iC_I$ and $D = D_R + iD_I$. The 2×2 matrix of Eq. (13) can be solved exactly to yield

$$\tan \phi = \frac{C_R D_I - C_I D_R}{C_R(2A - D_R) - C_I D_I}, \quad (21)$$

$$\tan 2\theta = \frac{2[4A^2 C_R^2 - 4AC_R(C_R D_R + C_I D_I) + (C_R^2 + C_I^2)(D_R^2 + D_I^2)]^{1/2}}{2AD_R - (D_R^2 + D_I^2)}, \quad (22)$$

with

$$e^{i\alpha'_2} M_2 = \cos^2 \theta A + 2 \sin \theta \cos \theta e^{i\phi} C + \sin^2 \theta e^{2i\phi} (A - D), \quad (23)$$

$$e^{i\alpha'_3} M_3 = \cos^2 \theta (A - D) - 2 \sin \theta \cos \theta e^{-i\phi} C + \sin^2 \theta e^{-2i\phi} A. \quad (24)$$

The corresponding U' elements are

$$U'_{e1} = \sqrt{\frac{2}{3}}, \quad U'_{e2} = \frac{\cos \theta}{\sqrt{3}}, \quad U'_{e3} = -\frac{\sin \theta}{\sqrt{3}} e^{-i\phi}, \quad (25)$$

$$U'_{\mu 3} = -\frac{\cos \theta}{\sqrt{2}} - \frac{\sin \theta}{\sqrt{3}} e^{-i\phi}, \quad U'_{\tau 3} = \frac{\cos \theta}{\sqrt{2}} - \frac{\sin \theta}{\sqrt{3}} e^{-i\phi}. \quad (26)$$

If we absorb the scale factor $\lambda_5 h^2 v^2 / 8\pi^2$ into the parameters A, C, D as well as m_0 , then the mass eigenvalues of Eq. (11) are given by

$$m'_k = \frac{1}{M_k} \left[\ln \frac{M_k^2}{m_0^2} - 1 \right], \quad (27)$$

which are the ones used in Eqs. (14) and (18). Since m_0 is an unknown, having to do with the dark-matter scalar mass, we fix it by requiring $M_1/m_0 = 10$, where $M_1 = |A + D|$. If we input the five parameters A, C_R, C_I, D_R, D_I , we will obtain $m'_{1,2,3}$ as well as the three mixing angles and the three CP phases. For our numerical analysis, we set

$$\Delta m_{21}^2 = 7.59 \times 10^{-5} \text{ eV}^2, \quad \Delta m_{32}^2 = 2.45 \times 10^{-3} \text{ eV}^2, \quad (28)$$

and vary θ_{13} in the range

$$\sin^2 2\theta_{13} = 0.05 \text{ to } 0.15. \quad (29)$$

Following Ref. [9], we look for solutions with $\sin^2 2\theta_{23} = 0.92$ and 0.96 . Whereas only normal hierarchy is allowed in the model of Ref. [9], we find solutions for both normal and inverted hierarchies, as well as quasi-degenerate solutions, as detailed below.

The predictions of this model regarding mixing angles are basically the same as in Ref. [9] for the special case of $b = 0$ there. Using Eqs. (19), (25), and (26), we find

$$\tan^2 \theta_{12} = \frac{1 - 3 \sin^2 \theta_{13}}{2}, \quad (30)$$

$$\tan^2 \theta_{23} = \frac{\left(1 - \frac{\sqrt{2} \sin \theta_{13} \cos \phi}{\sqrt{1 - 3 \sin^2 \theta_{13}}}\right)^2 + \frac{2 \sin^2 \theta_{13} \sin^2 \phi}{1 - 3 \sin^2 \theta_{13}}}{\left(1 + \frac{\sqrt{2} \sin \theta_{13} \cos \phi}{\sqrt{1 - 3 \sin^2 \theta_{13}}}\right)^2 + \frac{2 \sin^2 \theta_{13} \sin^2 \phi}{1 - 3 \sin^2 \theta_{13}}}. \quad (31)$$

The conventionally defined Dirac CP phase is given by $\delta_{CP} = \phi + \alpha'_3/2$, where α'_3 is defined in Eq. (18) and depends on the specific values of Eq. (9). For $\sin \theta_{13} = 0.16$, corresponding to $\sin^2 2\theta_{13} = 0.1$, this predicts $\tan^2 \theta_{12} = 0.46$. If $\text{Im}(C) = 0$, then $\delta_{CP} = \alpha'_3 = 0$, so this would predict $\sin^2 2\theta_{23} = 0.80$ which is of course ruled out. Using $\sin^2 2\theta_{23} > 0.92$, we find in this case $|\tan \phi| > 1.2$.

For each of the two values $\sin^2 2\theta_{23} = 0.92$ and 0.96 , we obtain 5 representative solutions, all as functions of $\sin^2 2\theta_{13}$. Using Eq. (30), we plot $\sin^2 2\theta_{12}$ versus $\sin^2 2\theta_{13}$ in Fig. 2. The

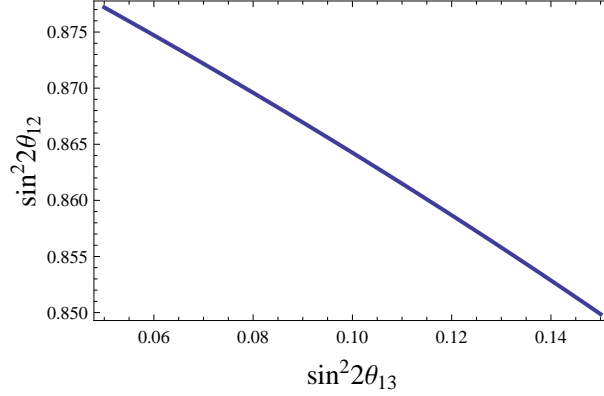


Figure 2: $\sin^2 2\theta_{12}$ versus $\sin^2 2\theta_{13}$.

characteristic features of the 5 solutions are listed in Table 2. For $Im(D) = 0$, we find one

solution	$Im(D)$	class	$ \tan \delta_{CP} $	m_{ee}
I	0	IH	2.05	0.020
II	$Re(D)$	IH	4.64	0.022
III	0	NH	3.59	0.002
IV	0	QD	2.20	0.046
V	$Re(D)$	QD	1.84	0.051

Table 2: Five representative solutions. Three have $Im(D) = 0$, and two have $Im(D) = Re(D)$. NH denotes normal hierarchy of neutrino masses, IH inverted, and QD quasi-degenerate. The values of $|\tan \delta_{CP}|$ and m_{ee} (in eV) are for $\sin^2 2\theta_{23} = 0.96$ and $\sin^2 2\theta_{13} = 0.10$.

solution for inverted ordering of neutrino masses, and two solutions for normal ordering (one of which is quasi-degenerate). For $Im(D) = Re(D)$, we again find one solution for inverted ordering, but the only solution for normal ordering is quasi-degenerate.

In Fig. 3 we show the physical neutrino masses $m'_{1,2,3}$ and the effective mass in neutrinoless double beta decay m_{ee} (in eV) as well as the model parameters (in eV^{-1}) for solution (I) in the case $\sin^2 2\theta_{23} = 0.96$. In Figs. 4-7 we show the same quantities for solutions (II),(III),(IV),(V)

in the cases of $\sin^2 2\theta_{23} = 0.92, 0.96, 0.92, 0.96$ respectively. Finally we show in Fig. 8 the values of $|\tan \delta_{CP}|$ for all 5 solutions in the case of $\sin^2 2\theta_{23} = 0.92$. It is clear that at $\sin^2 2\theta_{13} = 0.10$, large $|\tan \delta_{CP}|$ is predicted.

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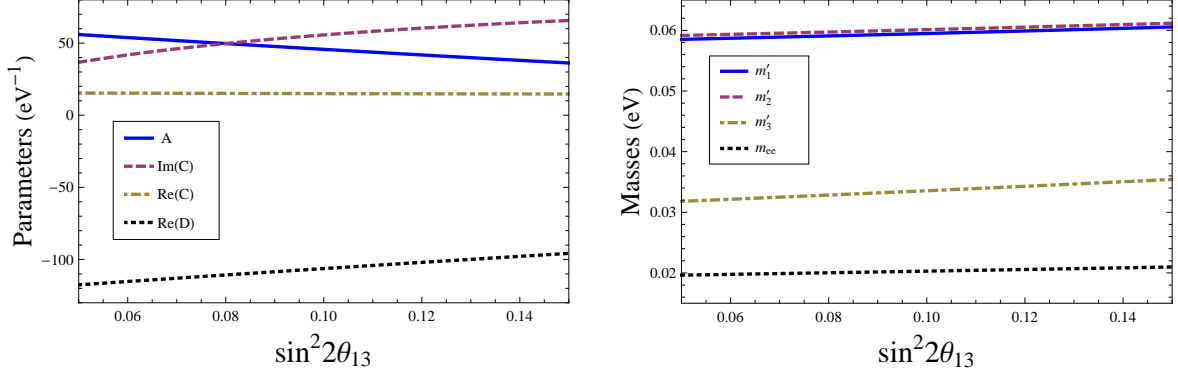


Figure 3: A_4 parameters and the physical neutrino masses and effective neutrino mass m_{ee} in neutrinoless double beta decay for the inverted hierarchy with $\text{Im}(D)=0$ and $\sin^2 2\theta_{23} = 0.96$.

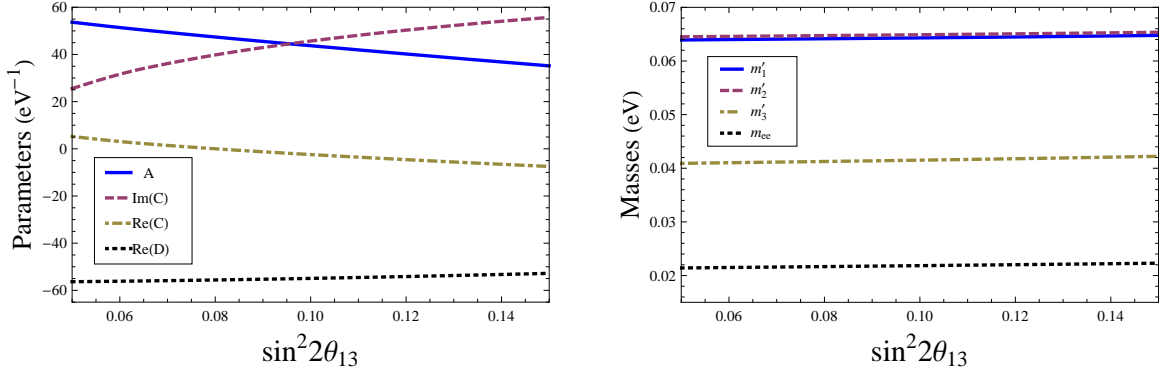


Figure 4: A_4 parameters and the physical neutrino masses and effective neutrino mass m_{ee} in neutrinoless double beta decay for the inverted hierarchy with $\text{Im}(D)=\text{Re}(D)$ and $\sin^2 2\theta_{23} = 0.92$.

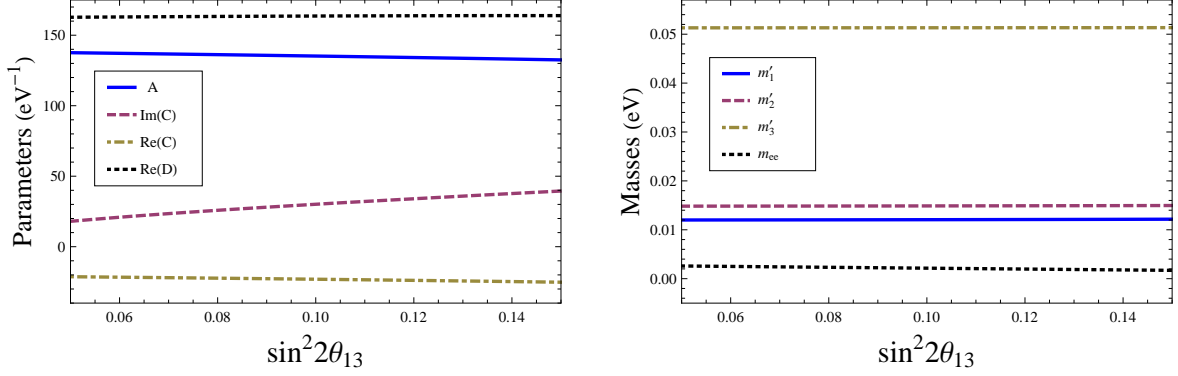


Figure 5: A_4 parameters and the physical neutrino masses and effective neutrino mass m_{ee} in neutrinoless double beta decay for the normal hierarchy with $\text{Im}(D)=0$ and $\sin^2 2\theta_{23} = 0.96$.

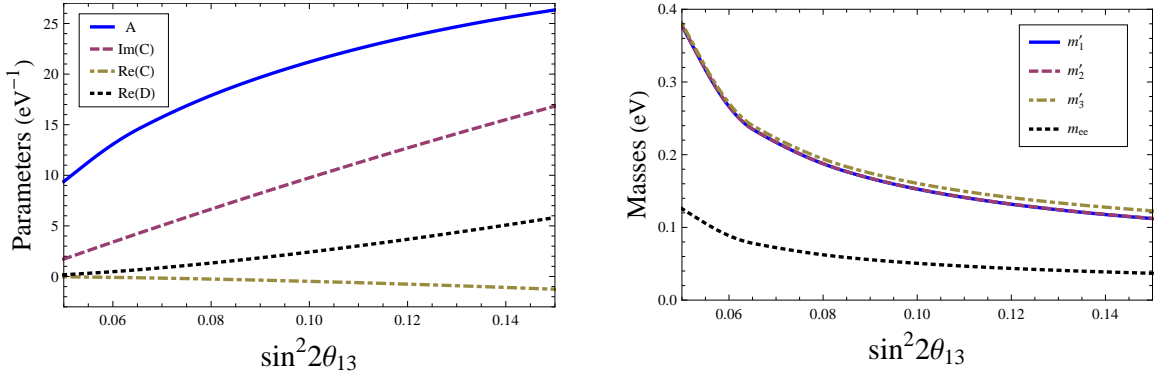


Figure 6: A_4 parameters and the physical neutrino masses and effective neutrino mass m_{ee} in neutrinoless double beta decay for quasi-degenerate neutrino masses with $\text{Im}(D)=0$ and $\sin^2 2\theta_{23} = 0.96$.

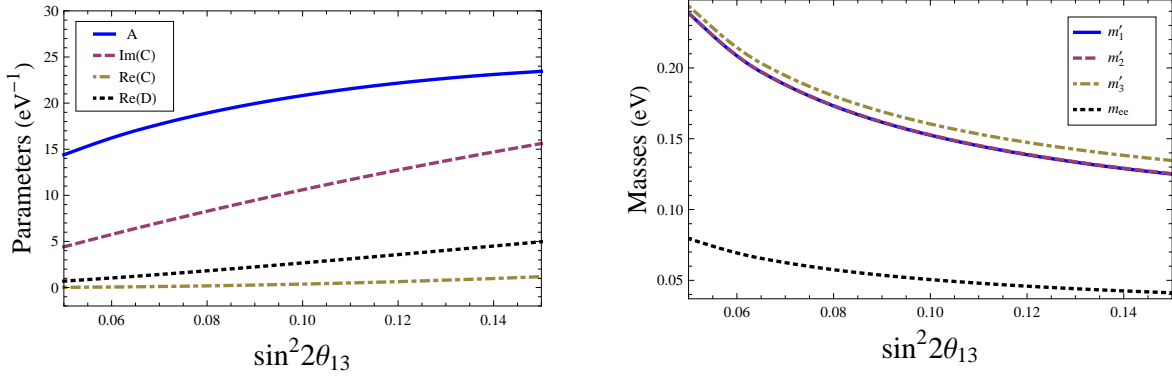


Figure 7: A_4 parameters and the physical neutrino masses and effective neutrino mass m_{ee} in neutrinoless double beta decay for quasi-degenerate neutrino masses with $\text{Im}(D)=\text{Re}(D)$ and $\sin^2 2\theta_{23} = 0.96$.

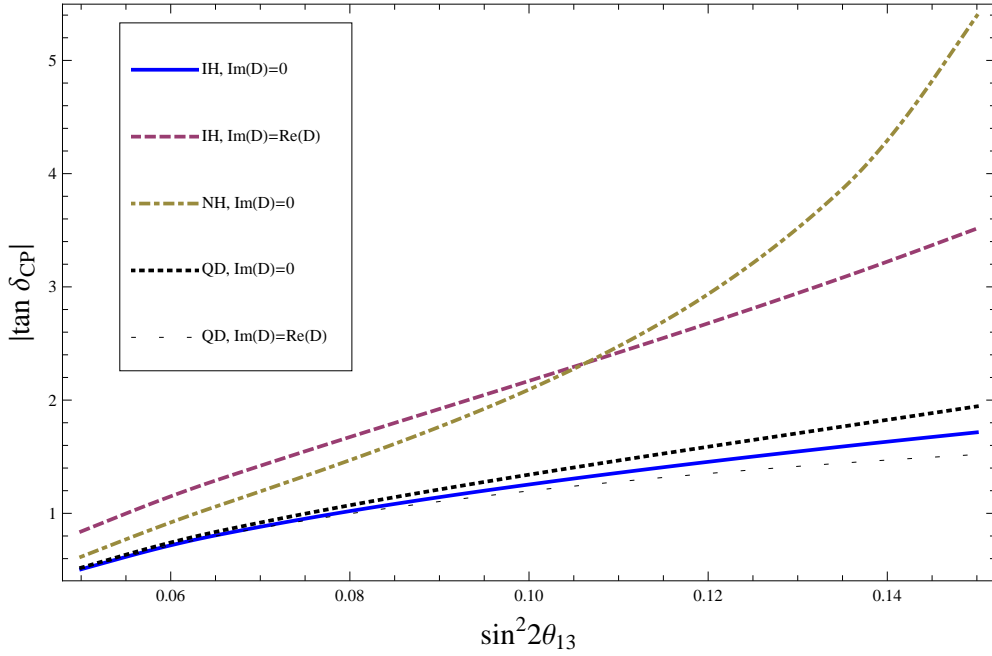


Figure 8: $|\tan \delta_{CP}|$ versus $\sin^2 2\theta_{13}$ for $\sin^2 2\theta_{23} = 0.92$.